Workshop on Non-Equilibrium Flow Phenomena in Honor of Mikhail Ivanov's 70th Birthday Novosibirsk, Russia, June 15-18, 2015

Continuum Regions of the Flow in the Evaporation-Condensation Problems

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MOTIVATION

The evaporation-condensation problem is a traditional problem of rarefied gas dynamics. Though the flow in the Knudsen layers near condensing-evaporating surfaces requires kinetic treatment, the applicability of continuum approach based on the Navier-Stokes or Euler equations for the description of some regions of the considered flow is of interest. Similar problems were also studied by

[1]. R.H. Edwards & R.L. Collins, RGD6, 1969.

[2]. A.K. Rebrov, M.Yu. Plotnikov & N.M. Bulgakova, RGD20, 1997.

[3]. I.A. Kuznetsova, A.A. Yushkanov, Yu.I. Yalamov // *High Temperature*, 38, 780-785 (2000).

The Navier-Stokes equations for steady plane flow

$$\rho u = const$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = \frac{4}{3}\frac{d}{dx}(\mu \frac{du}{dx})$$

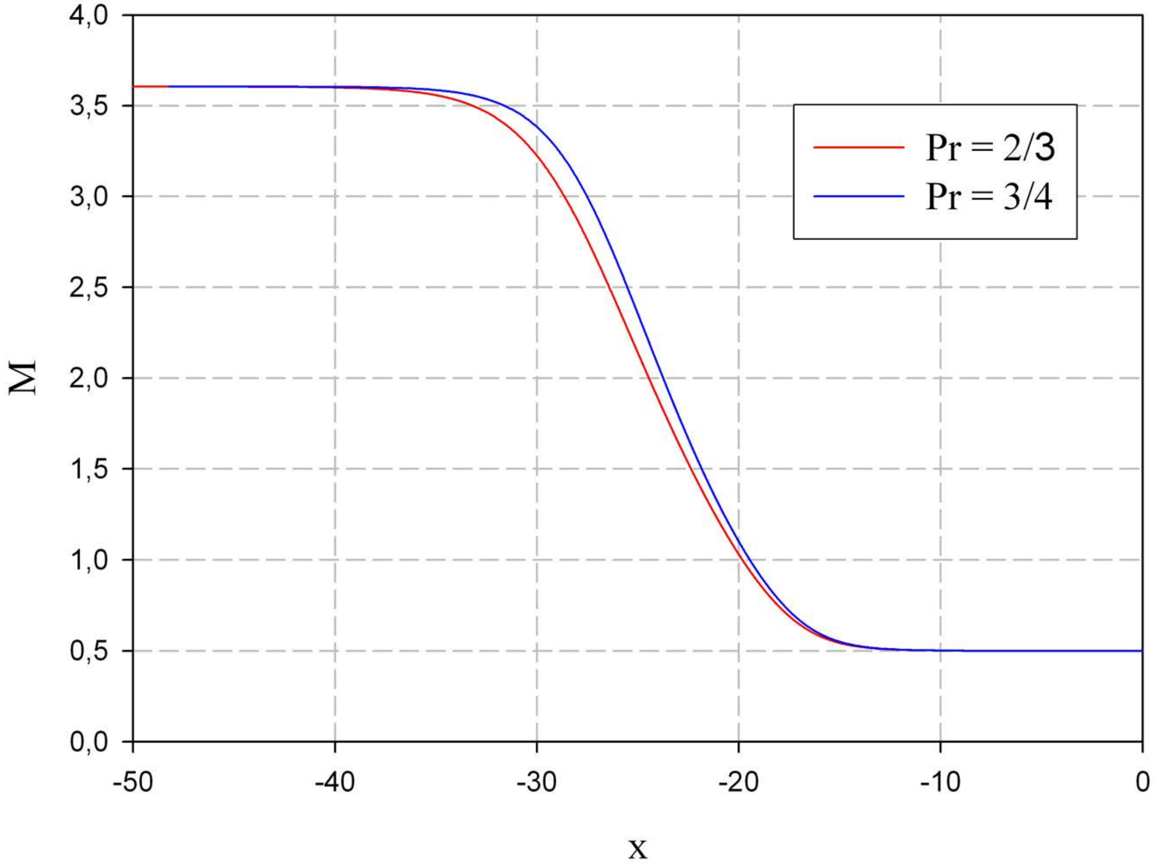
$$\rho u^2 + p - \frac{4}{3}\mu \frac{du}{dx} = C$$

$$\rho u \frac{d}{dx}(C_p T + \frac{u^2}{2}) = \lambda \frac{dT}{dx} + \frac{4}{3}\mu u \frac{du}{dx}$$

$$\Pr = \frac{\mu C_p}{\lambda} \qquad \Pr = \frac{2}{3} \qquad \Pr = \frac{3}{4} \qquad M_1 = u_1 / \sqrt{\kappa R T_1}$$

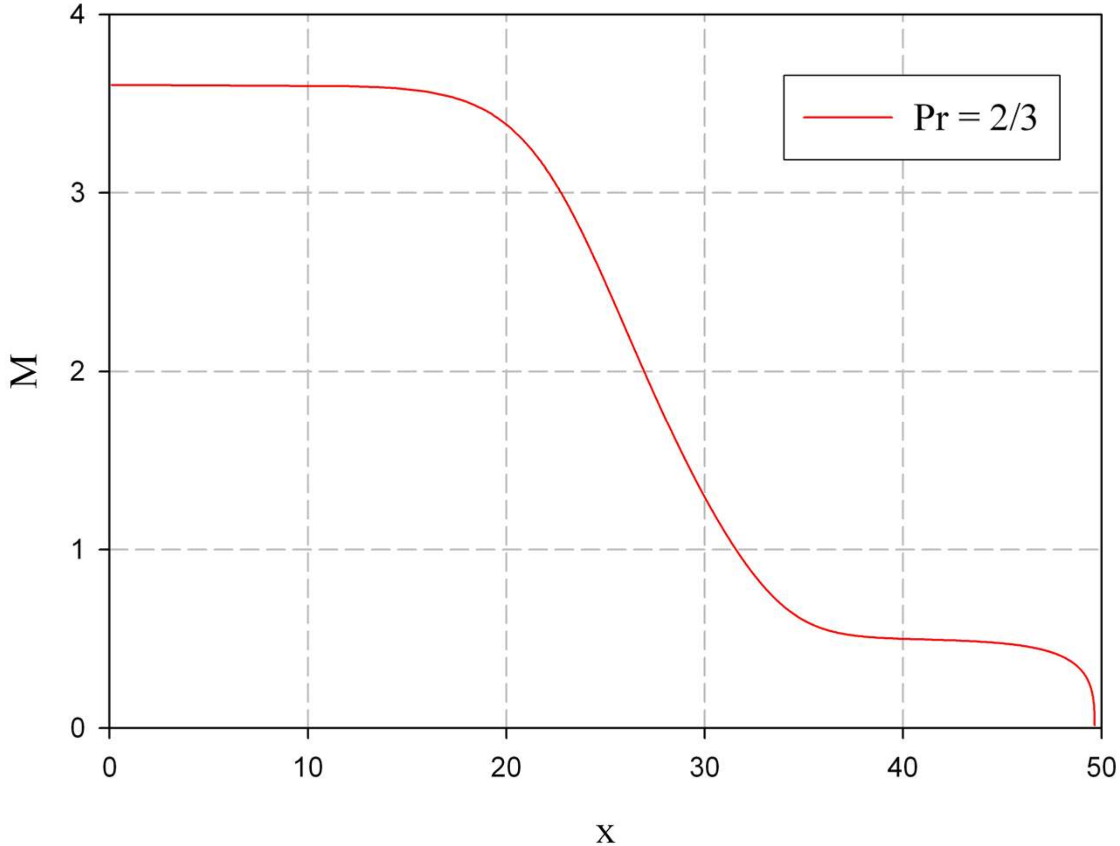
Only monatomic gas was considered (κ = 5/3) with $\mu \propto \sqrt{T}$ that corresponds to the hard spheres molecular model. The equations were solved by marching procedure using Runge-Kutta method. The upstream simulation was found to provide more stable solution compared to downstream one. All the solutions are completely determined by *C* and M_1.

Solutions of the Navier-Stokes equations for steady plane flow



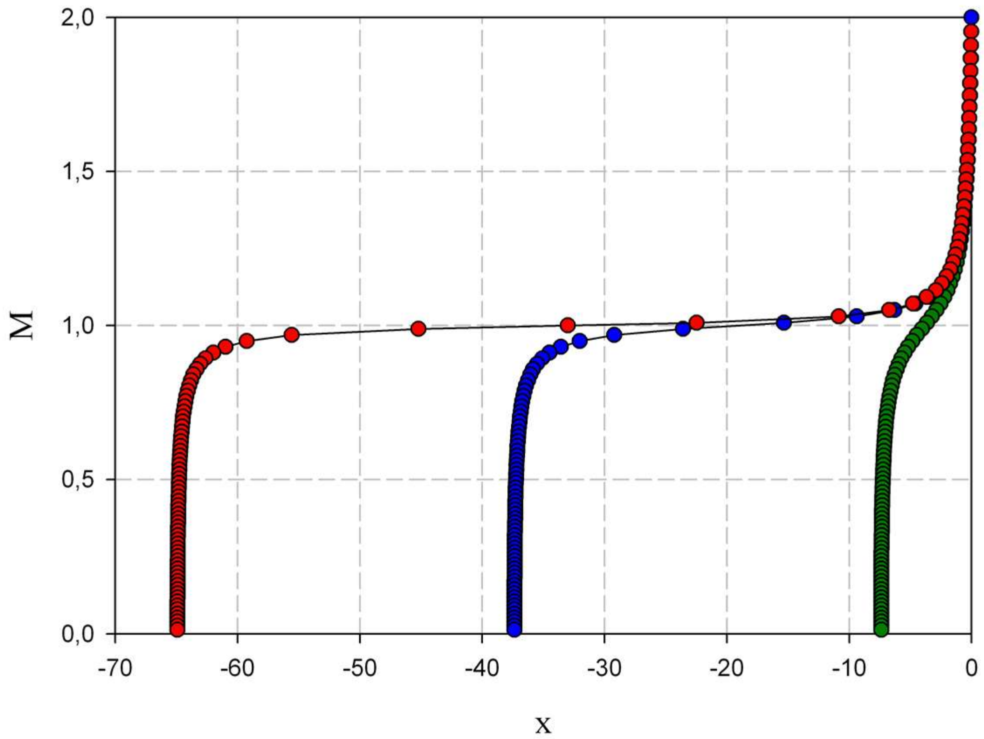
The shock wave structure for two values of Prandtl number (M_1 = 0.5, upstream simulation, with coordinate *x* being expressed in the units $4\mu_1\kappa M_1^2/3\rho_1 u_1$).

Solutions of the Navier-Stokes equations for steady plane flow



The shock wave structure for Pr = 2/3 (downstream simulation). Strange behavior of the Mach number in the subsonic region may correspond to supersonic condensation with the formation of the shock wave near the condensing surface.

Solutions of the Navier-Stokes equations for steady plane flow



The solution resembling the flow between evaporating and condensing surfaces for $M_1 = 2$, Pr = 2/3 and three values of constant C determining the different Reynolds numbers (upstream simulation). The shown profiles represent the exact solution of the Navier-Stokes equations, to determine what part of these profiles realized at the real flow between evaporating and condensing surfaces the boundary conditions at these surfaces should be used.

The boundary conditions

In this approach the distribution function in the gas flow at the surface is assumed to be of the Navier-Stokes type

$$f = f_0\{1 - \frac{1}{2pRT}\frac{4}{3}\mu\frac{du}{dx}[(\xi_x - u)^2 - \frac{\xi_y^2 + \xi_z^2}{2}] + \frac{1}{pRT}\lambda\frac{dT}{dx}(\xi_x - u)[1 - \frac{(\xi_x - u)^2 + \xi_y^2 + \xi_z^2}{5RT}]\}$$

with $u > 0$ at evaporating surface and $u < 0$ at condensing surface.

$$Q_i = \int_{-\infty}^{\infty} d\xi_y \int_{-\infty}^{\infty} d\xi_z \int_{0}^{\infty} \varphi_i f d\xi_x \qquad \varphi_i = m\xi_x^i,\ i = 0, 1, 2 \qquad \varphi_3 = m\xi_x(\xi_x^2 + \xi_y^2 + \xi_z^2)/2$$

To find the approximate solution of the system of equations $Q_i = Q_{iw}$ ($i = 0, 1, 2, 3$) where Q_{iw} represent the corresponding moments obtained for half-space Maxwellian distribution function based on the parameters p_w, T_w on the surface, the minimization of the functional

$$F(p_w, T_w) = \sum_{i=0}^{3} \frac{(Q_i - Q_{iw})^2}{Q_i^2}$$

by the least squares method was performed that allows one to find unknowns (p_w, T_w).

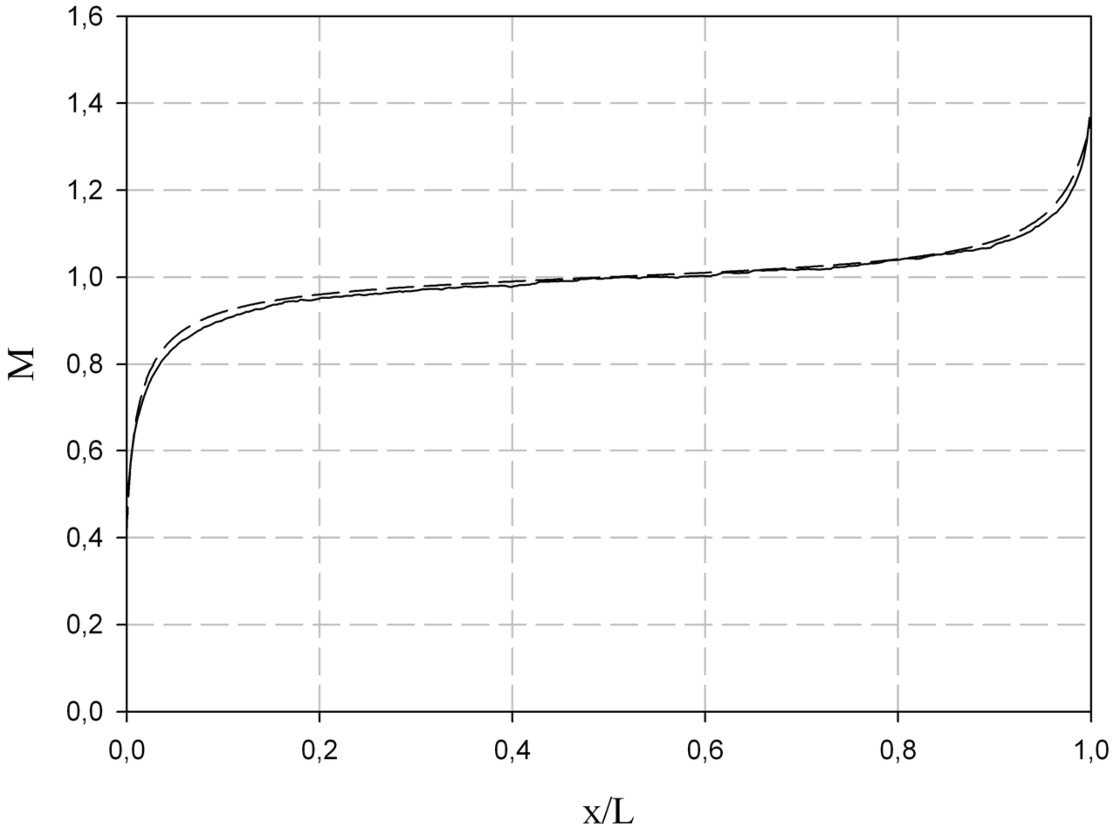
The position of evaporating and condensing surfaces

The right boundary of the profile corresponding to the position of the fully condensing surface was found from the condition $Q_1 = 0$ with $u < 0$ (no evaporating flux). The value of the Mach number at this surface appeared to be M = 1.364 that is in good agreement with the results obtained by DSMC simulation (see below).

The left boundary of the profile corresponding to the position of the evaporating surface was found by minimization of the functional $F(p_w, T_w)$ with $u > 0$. The value of the Mach number at this surface appeared to be M = 0.409 that is in agreement with the values 0.423 [1] and 0.386 [3] obtained for strong evaporation into vacuum.

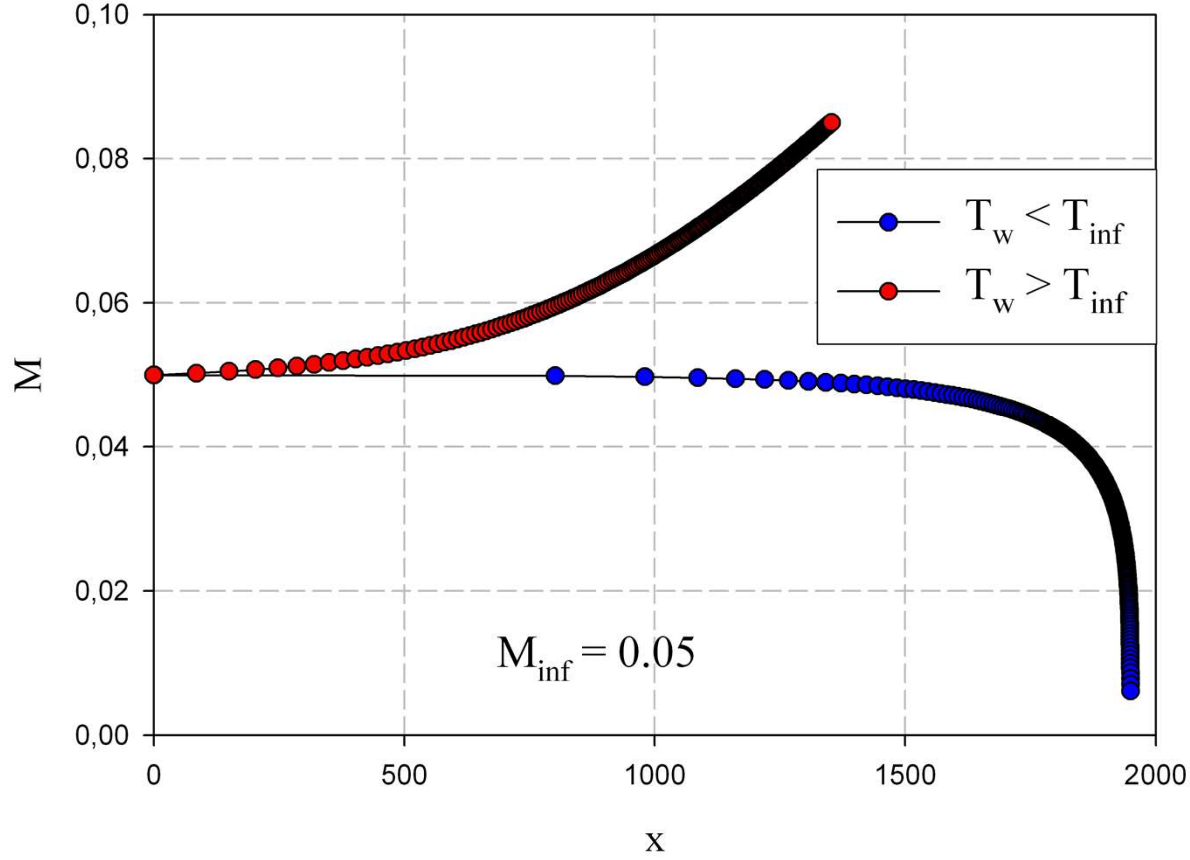
The relative back flux to the evaporating surface appeared to be 0.196 while the prediction of continuum theory for this value is 0.184 which are in good agreement between each other and with the value 0.189 reported in [1].

The plane flow between evaporating and condensing surfaces



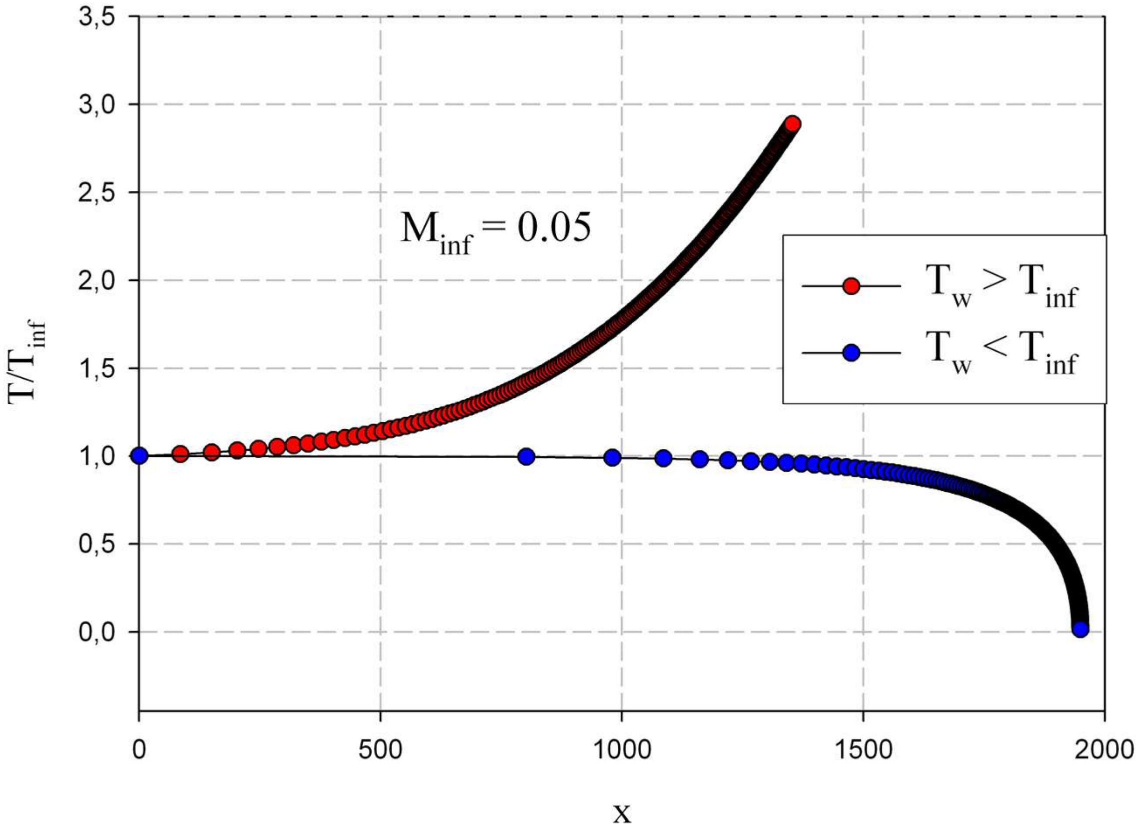
The Mach number profiles for the flow between evaporating and condensing surfaces for Kn = l_0/L = 0.00625 (solid line – DSMC solution, dashed line – continuum solution). Note that the position of the point with M = 1 is very close to the middle of the distance between the surfaces in both approaches.

Solutions of the Navier-Stokes equations for steady plane flow



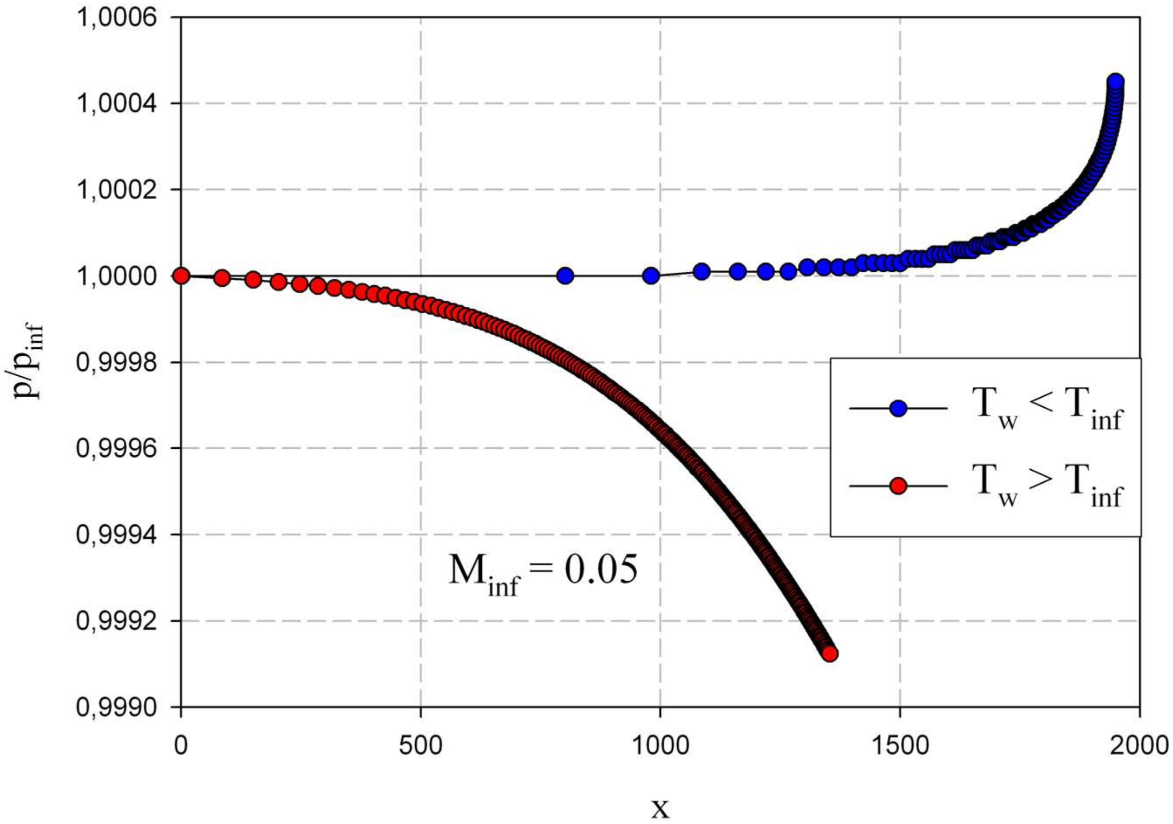
The Mach number profiles resembling the flow at subsonic condensation for two ranges of surface temperature (downstream simulation; note that the condensing surface may be placed at each point of the curves).

Solutions of the Navier-Stokes equations for steady plane flow



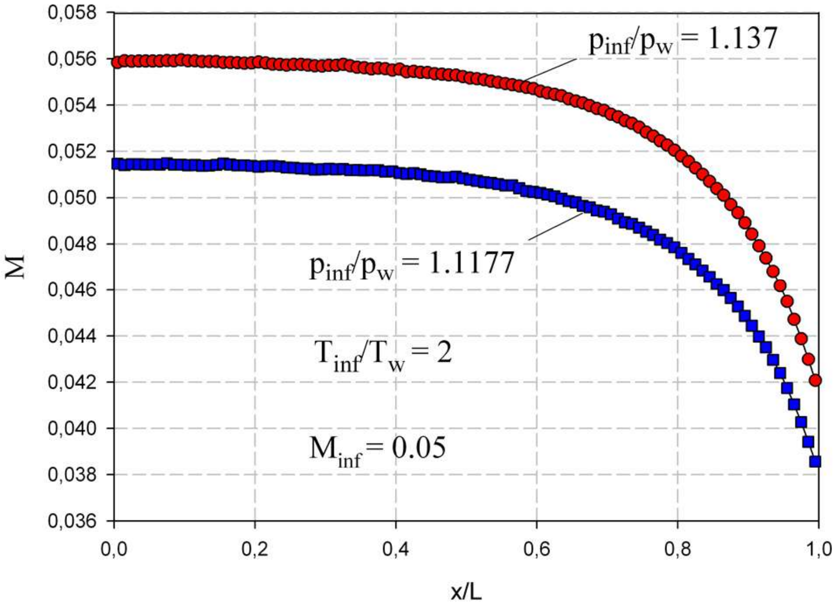
The temperature profiles in the flow at subsonic condensation for two ranges of surface temperature.

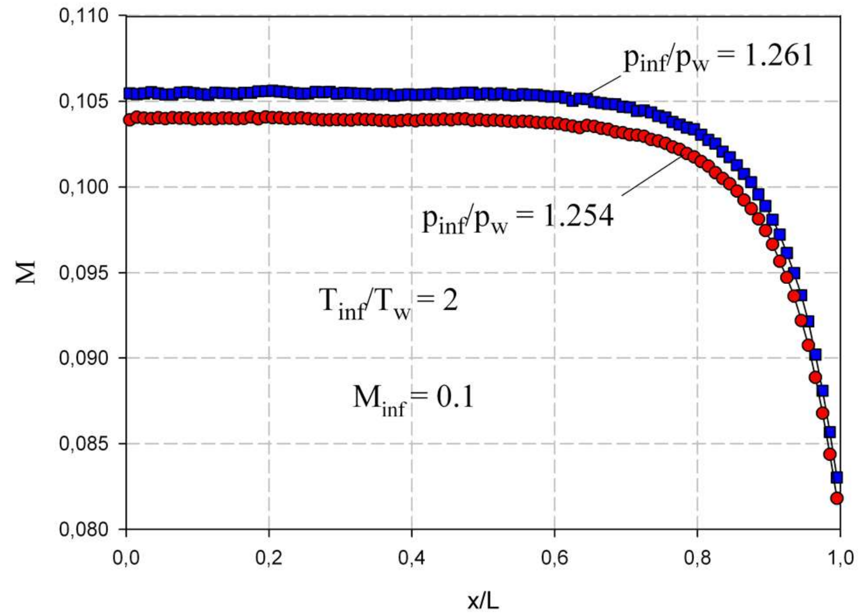
Solutions of the Navier-Stokes equations for steady plane flow



The pressure profiles in the flow at subsonic condensation for two ranges of surface temperature (note that the flow is practically isobaric, that is in agreement with available data (C. Cercignani, RGD18, 1994)).

The subsonic condensation





The distribution of Mach number near condensing surface for $T_{inf}/T_w = 2$, $M_{inf} = 0.05$ (left) and $M_{inf} = 0.1$ (right) obtained by DSMC method for HS molecular model and Kn = 0.01 for the values of p_{inf}/p_w reported by Soga (RGD20, 1991, red symbols) and predictions of Navier-Stokes equations obtained by minimization of the functional $F(p_w, T_w)$ with $u < 0$ (blue symbols).

Note, that in spite of practically isobaric flow the pressure at the surface (p_w) for $M_{inf} = 0.05$ is lower compared to the pressure in the flow by about 10%. The more the Mach number M_{inf} the more the pressure jump at the surface.

Evaporation from radial sources into vacuum

Cylindrical source

$$\rho u r = const$$

$$\rho u \frac{du}{dr} + \frac{dp}{dr} = \frac{4}{3}\frac{d\varphi}{dr} + \frac{8}{3}\frac{\varphi}{r} + \frac{2}{3}\frac{u}{r}\frac{d\mu}{dr}$$

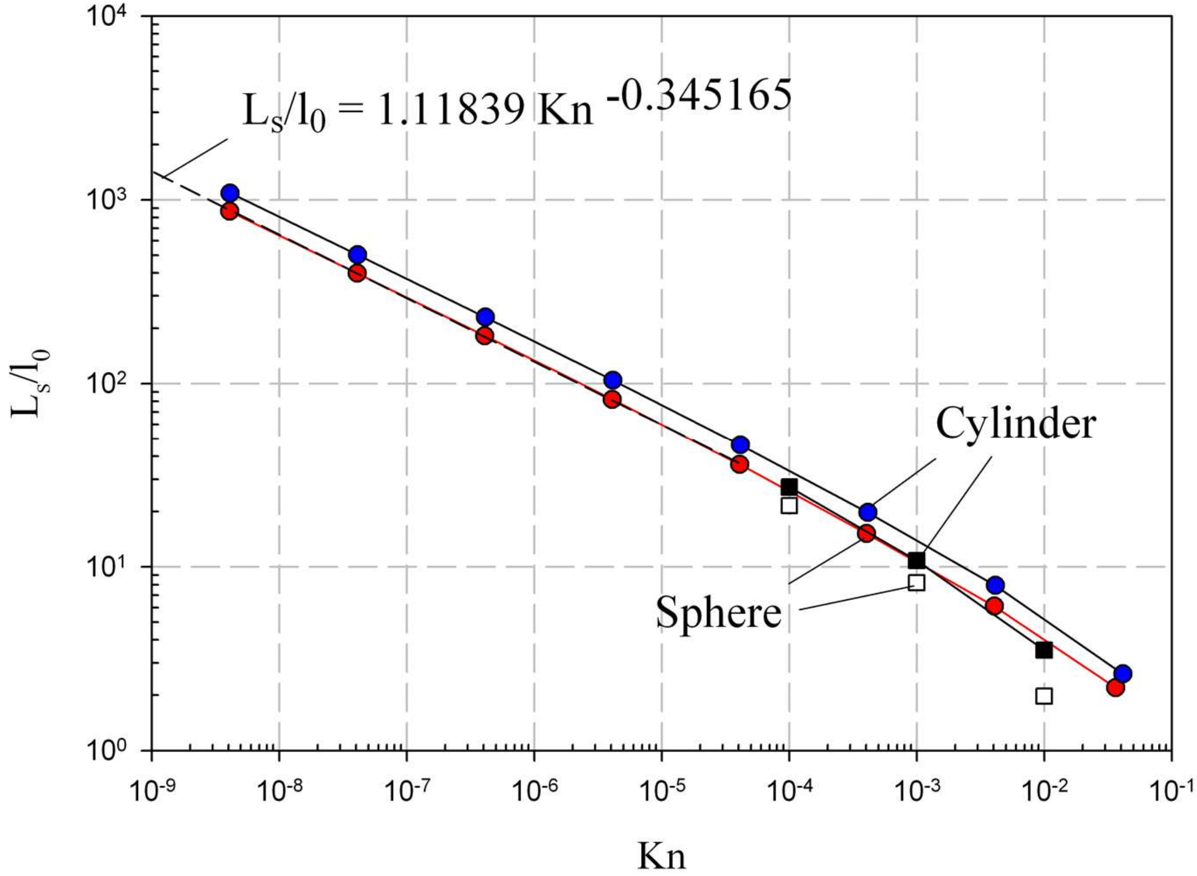
Spherical source

$$\rho u r^2 = const$$

$$\rho u \frac{du}{dr} + \frac{dp}{dr} = \frac{4}{3}\frac{d\varphi}{dr} + \frac{4\varphi}{r}$$

$$C_P T + \frac{u^2}{2} = const \qquad \varphi \equiv \mu\left(\frac{du}{dr} - \frac{u}{r}\right) \qquad \mu \propto \sqrt{T}$$

Transonic region at strong evaporation from radial sources



Radial size of the transonic region for strong evaporation into vacuum from cylindrical and spherical sources obtained by upstream simulation versus Knudsen number (circles – this study, squares – DSMC data by Rebrov, Plotnikov and Bulgakova, 20 RGD, 1997, dashed line – approximation for spherical source for small Knudsen number range ($Kn < 10^{-4}$)).

An extremely large effect of the evaporating surface curvature on the transonic region size

$$\frac{L_s}{l_0} = 1.11839 Kn^{-0.345165} \qquad \frac{L_s}{R} = 1.11839 Kn^{0.654835}$$

The estimation of the transonic region size for high Reynolds numbers as $L_s / R \propto \mathrm{Re}^{-2/3}$ was earlier discovered by A. Sakurai (1958) and R.H. Edwards & R.L. Collins (1969).

$$Kn = \frac{l_0}{R} \propto \frac{10^{-5}}{6.4 \cdot 10^{8}} = 1.56 \cdot 10^{-14}$$

$$\frac{L_s}{l_0} = 5.82 \cdot 10^{4} \qquad L_s = 0.58 cm \qquad \frac{L_s}{R} \propto 10^{-9}$$

Considering the problem of strong evaporation into vacuum an extremely large effect of the evaporating surface curvature on the size of the transonic region was discovered. Thus for spherical evaporating surface with radius about 6400 km (i. e. the Earth radius) the radial size of the transonic region at atmospheric conditions is about 1 cm, while for plane evaporating surface the transonic region is infinitely large.

CONCLUSION

- **The upstream simulation of the Navier-Stokes equations for steady plane and radial flows was found to provide more stable solution compared to downstrean one that is an interesting feature of these equations.**
- **The new approach for obtaining the boundary conditions on evaporating and condensing surfaces based on approximate equilibration of four lowest moments of the Navier-Stokes distribution function to corresponding moments of the Maxwellian half-space distribution function by least squares method was tested and found to provide quite reasonable results for both strong evaporation and weak subsonic condensation regimes.**
- **The Navier-Stokes equations provide good description of subsonic condensation with low Mach numbers ($M_{inf} \leq 0.05$). For this flow the boundary condition on the surface (p_w, T_w) obtained by the new approach are even in better agreement with DSMC data compared to the available data obtained by the model kinetic equation. The Navier-Stokes equations surely failed to describe the subsonic condensation with moderate Mach numbers ($M_{inf} > 0.5$).**

- **In accordance with the predictions of the Navier-Stokes equations the weak subsonic condensation takes place at practically isobaric conditions that is in agreement with available data, though the pressure at the surface (p_w) for $M_{inf} = 0.05$ is lower compared to the pressure in the flow by about 10% that reveals the pressure jump at the condensing surface.**
- **For the flow between evaporating and condensing surfaces the Navier-Stokes equations predict the position of the point with M = 1 very close to the middle of the distance between the surfaces.**
- **Considering the problem of strong evaporation into vacuum an extremely large effect of the evaporating surface curvature on the size of the transonic region was discovered. Thus for spherical evaporating surface with radius about 6400 km (i. e. the Earth radius) the radial size of the transonic region at atmospheric conditions is about 1 cm, while for plane evaporating surface the transonic region is infinitely large.**
- **The quality of the description of the flows with evaporation and condensation including the regions in the Knudsen layers by continuum approach proved to be higher than it may be expected taking into account the discontinuity of the distribution function on the surface with phase transition.**

Thank you for your attention!